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Propagators for Driven Coupled Harmonic Oscillators

by

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Propagators for driven coupled harmonic oscillators

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1. Introduction

Although the Feynman path-integral formulation[1] offers a general approach for treating quantum-mechanical systems, only several problems can be solved exactly. Two of these are the driven harmonic oscillator with a quadratic Hamiltonian[2] and the time-dependent damped driven harmonic oscillator.[3] A number of situations such as superconducting quantum interference devices,[4] quantum nondemolition measurments,[5] magnetohydrodynamics,[6] etc., can be described by driven copuled harmonic oscillators. Introducing the Caldirola-Kanai Hamiltonian,[7] one can obtain the time-dependent Schroedinger equation for the damped harmonic oscillator. However, it has been a matter of debate as to whether or not this Schroedinger equation represents the quantum mechanical dissipative system.[8] Some workers[9] claim affirmation while others[10] object to it. This problem has been critically reviewed by Greenberger[11] and Cervero and Villaroel.[12]

The purpose of this paper is to derive the propagator for a driven coupled harmonic oscillators (DCHO) system from our previous work[13] for both coupled and coupled driven harmonic oscillators by means of the path-integral method. We introduce two harmonic oscillators that are coupled together with another spring. We review the classical case and construct the form of the propagator for DCHO, respectively, in Secs. 2 and 3. Section 4 gives the exact derivation of the propagator for the coupled harmonic oscillators (CHO), and in Sec. 5 we evaluate the exact propagator for DCHO by using the results obtained in Sec. 4. The energy expectation values of CHO are evaluated in Sec. 6, and finally we give results and discussion in Sec. 7.

2. Classical case

In this section we consider a system of two harmonic oscillators which are coupled together by means of another spring. We assume that the masses of the oscillators and three spring constants are all the same. Let the forces $f_1(t)$ and $f_2(t)$ exerted on the two oscillators and their displacements be x_1 and x_2. Then the Hamiltonian for DCHO can be written as

$$H = \frac{1}{2m}(p_1^2+p_2^2) + m\omega^2(x_1^2-x_1x_2+x_2^2) - f_1(t)x_1 - f_2(t)x_2 \quad , \tag{2.1}$$

where $\omega^2 = k/m$. Hamilton's equations of motion for Eq. (2.1) are

$$\dot{x}_1 = p_1/m \tag{2.2}$$

$$\dot{x}_2 = p_2/m \tag{2.3}$$

$$\dot{p}_1 = m\omega^2(x_2-2x_1) + f_1(t) \tag{2.4}$$

$$\dot{p}_2 = m\omega^2(x_1-2x_2) + f_2(t) \tag{2.5}$$

Equations (2.1)-(2.5) yield the Lagrangian,

$$\begin{aligned} L &= (p_1\dot{x}_1+p_2\dot{x}_2) - H \\ &= \frac{m}{2}(\dot{x}_1^2+\dot{x}_2^2) - m\omega^2(x_1^2-x_1x_2+x_2^2) + f_1(t)x_1 + f_2(t)x_2 \quad , \end{aligned} \tag{2.6}$$

with the corresponding equations of motion

$$x_1 + \omega^2(2x_1 - x_2) = f_1(t)/m \tag{2.7}$$

$$x_2 + \omega^2(2x_2 - x_1) = f_2(t)/m \quad . \tag{2.8}$$

The classical solutions of Eqs. (2.7) and (2.8) are given by

$$\begin{aligned} x_1(t) &= A\sin(\omega t) + B\cos(\omega t) + C\sin(\sqrt{3}\omega t) + D\cos(\sqrt{3}\omega t) \\ &\quad + \int^t d\tau \int^\tau d\nu \; e^{i\omega(2\tau-\nu-t)} \; [f_1(\nu) + f_2(\nu)] \quad , \end{aligned} \tag{2.9}$$

and

$$\begin{aligned} x_2(t) &= A\sin(\omega t) + B\cos(\omega t) - C\sin(\sqrt{3}\omega t) - D\cos(\sqrt{3}\omega t) \\ &\quad + \int^t d\tau \int^\tau d\nu \; e^{i\omega(2\tau-\nu-t)} \; [f_1(\nu) - f_2(\nu)] \quad . \end{aligned} \tag{2.10}$$

3. Path integral of driven coupled harmonic oscillators

In the path-integral formulation, the solution of the Schroedinger equation is given as the path-dependent integral equations with propagator K,

$$\psi(x_1, x_2, t) = \int dx_1' dx_2' \; K(x_1, x_2, t;\; x_1', x_2', 0) \; \psi(x_1', x_2', 0) \quad , \tag{3.1}$$

which gives the wavefunction $\psi(x_1, x_2, t)$ at time t in terms of the wave function $\psi(x_1', x_2')$ at time $t = 0$. The propagator in Eq. (3.1) can be written by means of the Feynman path integral

$$K(x_1,x_2,t;x_1',x_2',0) = \int_{(x_1',x_2',0)}^{(x_1,x_2,t)} Dx(t)\ \exp[(i/\hbar)S(x_1,x_2,x_1',x_2';\ t) \quad (3.2)$$

where

$$Dx(t) = \lim_{N\to\infty} \frac{1}{A} \prod_{j=1}^{N-1} [dx_{1j}dx_{2j}/A^2] \quad , \tag{3.3}$$

and $S(x_1,x_2,x_1',x_2';\ t)$ is the action defined as the time integral over the Lagrangian $L(\dot{x}_1,\dot{x}_2,x_1,x_2;\ t)$ between $t = t$ and $t = 0$:[1]

$$S(x_1,x_2,x_1',x_2';t) = \int_0^t dt\ L(\dot{x}_1,\dot{x}_2,x_1,x_2;\ t) \quad . \tag{3.4}$$

In Eq. (3.3) A is the normalization factor given by

$$A = [2\pi i\hbar\epsilon/m]^{\frac{1}{2}} \quad , \qquad \epsilon = \lim_{N\to\infty}(t/N) \quad . \tag{3.5}$$

Substituting Eq. (2.6) into Eq. (3.4), the action becomes

$$S(x_1,x_2,x_1',x_2';\ t) = S_c(x_1,x_2,x_1',x_2';\ t)$$

$$+ \int_0^t d\tau\ \frac{m}{2}\{\dot{y}_1^2(\tau) + \dot{y}_2^2(\tau) - 2\omega^2[y_1^2(\tau) - y_1(\tau)y_2(\tau) + y_2^2(\tau)]\} \quad , \tag{3.6}$$

where S_c is the classical action and y_i is the deviation of $x_i(t)$ from its classical path x_{ci} given as

$$y_i = x_i - x_{ci} \quad . \quad (i = 1,2) \tag{3.7}$$

Then the propagator [Eq. (3.2)] can be expressed as

$$K(x_1, x_2, t;\ x_1', x_2', 0) = F(t)\ e^{iS_c/\hbar} \quad . \tag{3.8}$$

Here, F(t) is the multiplicative function given in the form

$$F(t) = \int_0^0 Dx(t)\ \{\exp\ [(im/2\hbar)\int_0^t dt\ (\dot{y}_1^2 + \dot{y}_2^2 - 2\omega^2(y_1^2 - y_1 y_2 + y_2^2])\} \quad . \tag{3.9}$$

It is easy to show that F(t) has the same form for CHO and DCHO. Therfore, the propagator depends only on the classical action in both cases. In Eq. (3.9), change the variables $x_1 \pm x_2$ into

$$z_1 = \frac{1}{\sqrt{2}}(x_1 - x_2) \tag{3.10}$$

$$z_2 = \frac{1}{\sqrt{2}}(x_1 + x_2) \quad , \tag{3.11}$$

we can reduce the condition $(y_1, y_2) = (0,0)$ to $(z_1, z_2) = (0,0)$. Applying Eqs. (3.10) and (3.11) to Eq. (3.9), the multiplicative function becomes

$$F(t) = J\int_0^0 Dz(t)\ \{\exp[(im/2\hbar)\int_0^t Dz(t)\ [(\dot{z}_1^2 - \omega^2 z_1^2) + (\dot{z}_2^2 - 3\omega^2 z_2^2)]\} \quad , \tag{3.12}$$

$$
\begin{bmatrix} y_{11} \\ y_{12} \\ y_{21} \\ \cdot \\ \cdot \\ y_{N-1,2} \\ y_{N1} \\ y_{N2} \end{bmatrix}
=
\begin{bmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & , & 0 & 0 & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & , & 0 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & \cdot & \cdot & \cdot & , & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & , & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & , & \cdot & \cdot & \cdot \\
0 & 0 & 0 & \cdot & \cdot & \cdot & , & \frac{1}{\sqrt{2}} & 0 & 0 \\
0 & 0 & 0 & \cdot & \cdot & \cdot & , & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & 0 & 0 & \cdot & \cdot & \cdot & , & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\begin{bmatrix} z_{11} \\ z_{12} \\ z_{21} \\ \cdot \\ \cdot \\ z_{N-1,2} \\ z_{N1} \\ z_{N2} \end{bmatrix}
\tag{3.13}
$$

In Eq. (3.12), J becomes unity.

If the action is separated into the functionals with only same variables in the path integral, then this integral can be represented by the multiplication of path integrals with each variable. Therefore, Eq. (3.12) becomes

$$F(t) = F_1(t)\ F_2(t)$$

$$= \left\{\int_0^0 Dz_1(t)\ \exp[(im/2\hbar)\int_0^t dt\ (\dot{z}_1^2-\omega^2 z_1^2)]\right\}$$

$$\times \left\{\int_0^0 Dz_2(t)\ \exp[(im/2\hbar)\int_0^t dt\ (\dot{z}_2^2-3\omega^2 z_2^2)]\right\} \quad . \tag{3.14}$$

Since $F_1(t)$ and $F_2(t)$ are the path integrals of the harmonic oscillator, the evaluation of Eq. (3.14) gives

$$F(t) = \frac{m\omega}{2\pi i\hbar}\left[\frac{\sqrt{3}}{\sin(\omega t)\ \sin(\sqrt{3}\omega t)}\right]^{\frac{1}{2}} \quad . \tag{3.15}$$

Hence, the propagator of DCHO can be written as

$$K(x_1,x_2,t;\ x_1',x_2',0) = \frac{m\omega}{2\pi i\hbar}\left[\frac{\sqrt{3}}{\sin(\omega t)\ \sin(\sqrt{3}\omega t)}\right]^{\frac{1}{2}} e^{iS_c/\hbar} \quad . \tag{3.16}$$

4. **Propagator for the coupled harmonic oscillators**

To evaluate the exact propagator expressed by Eq. (3.16), we should first obtain the propagator for CHO. The classical action of CHO is

$$S_c = \int_0^t d\tau\ \left(\frac{m}{2}(\dot{x}_{c1}^2+\dot{x}_{c2}^2) - m\omega^2(x_{c1}^2-x_{c1}x_{c2}+x_{c2}^2)\right) \quad , \tag{4.1}$$

where x_{c1} and $\dot{x}_{c1}$ are the classical path and velocity, respectively. Integrating Eq. (4.1) over the time, we get

$$S_c = \frac{m}{2}(x_{c1}\dot{x}_{c1}+x_{c2}\dot{x}_{c2})\Big|_0^t - \int_0^t d\tau\ \frac{m}{2}\ x_{c1}\{\ddot{x}_{c1}+\omega^2(2x_{c1}-x_{c2})\}$$

$$- \int_0^t d\tau \, \frac{m}{2} \, x_{c2}(\ddot{x}_{c2}+\omega^2(2x_{c2}-x_{c1}))$$

$$= \frac{m}{2} \, [x_{c1}(t)\dot{x}_{c1}(t)+x_{c2}(t)\dot{x}_{c2}(t)-x_{c1}(0)\dot{x}_{c1}(0)-x_{c2}(0)\dot{x}_{c2}(0)] \quad . \quad (4.2)$$

Here the second and third terms become zero because of the equations of motion [see Eqs. (2.7) and (2.8)], given as

$$\ddot{x}_1 + \omega^2(2x_1-x_2) = 0 \quad , \qquad (4.3)$$

$$\ddot{x}_2 + \omega^2(2x_2-x_1) = 0 \quad . \qquad (4.4)$$

To obtain the exact expression of Eq. (4.2), we solve Eqs. (4.3) and (4.4) to obtain

$$x_1 = x_1(t) = A \sin(\omega t) + B \cos(\omega t) + C \sin(\sqrt{3}\omega t) + D \cos(\sqrt{3}\omega t) \qquad (4.5)$$

$$x_2 = x_2(t) = A \sin(\omega t) + B \cos(\omega t) - C \sin(\sqrt{3}\omega t) - D \cos(\sqrt{3}\omega t) \quad , \qquad (4.6)$$

and $\dot{x}_1$ and $\dot{x}_2$ are given, respectively, by

$$\dot{x}_1 = \dot{x}_1(t) = \omega(A \cos(\omega t) - B \sin(\omega t) + \sqrt{3}C \sin(\sqrt{3}\omega t) - \sqrt{3}D \sin(\sqrt{3}\omega t)) \qquad (4.7)$$

$$\dot{x}_2 = \dot{x}_2(t) = \omega(A \cos(\omega t) - B \sin(\omega t) - \sqrt{3}C \cos(\sqrt{3}\omega t) + \sqrt{3}D \sin(\sqrt{3}\omega t)) \quad . \qquad (4.8)$$

Equations (4.5)-(4.8) give

$$x_1' = x_1(0) = B + D \tag{4.9}$$

$$x_2' = x_2(0) = B - D \tag{4.10}$$

$$\dot{x}_1' = \dot{x}_1(0) = \omega(A + \sqrt{3}C) \tag{4.11}$$

$$\dot{x}_2' = \dot{x}_2(0) = \omega(A - \sqrt{3}C) \quad . \tag{4.12}$$

The time-dependent constants A, B, D and D obtained from Eqs. (4.5) and (4.6), and Eqs. (4.9) and (4.10) can be expressed as

$$A = [\frac{1}{2}\sin(\omega t)]\,\{x_1 + x_2 - (x_1' + x_2')\cos(\omega t)\} \tag{4.13}$$

$$B = \frac{1}{2}\,(x_1' + x_2') \tag{4.14}$$

$$C = [\frac{1}{2}\sin(\sqrt{3}\omega t)]\,\{x_1 - x_2 + (x_1' - x_2')\cos(\sqrt{3}\omega t)\} \tag{4.15}$$

$$D = \frac{1}{2}\,(x_1' - x_2') \quad . \tag{4.16}$$

Substitution of Eqs. (4.5) -(4.16) into (4.2) gives the classical action:

$$S_c = \frac{m\omega}{4}\,\{(x_1^2 + x_2^2 + x_1'^2 + x_2'^2)[\cot(\omega t) + \sqrt{3}\cot(\sqrt{3}\omega t)]\}$$

$$+ 2(x_1 x_2 + x_1' x_2')\,[\cot(\omega t) - \sqrt{3}\cot(\sqrt{3}\omega t)]$$

$$- 2(x_1x_1' + x_2x_2') \{[1/\sin(\omega t) + \sqrt{3}/\sin(\sqrt{3}\omega t)]\}$$

$$+ 2(x_1x_2' + x_2x_1') [-1/\sin(\omega t) + (\sqrt{3}/\sin(\sqrt{3}\omega t)] \quad . \tag{4.17}$$

Combining Eqs. (4.17) and (3.16), we obtain the propagator for CHO:

$$K(x_1,x_2,t;\ x_1',x_2',0) = \frac{m\omega}{2\pi i\hbar} [\sqrt{3}/\sin(\omega t)\ \sin(\sqrt{3}\omega t)]^{\frac{1}{2}}$$

$$\times \exp\{(im\omega/4\hbar)[(x_1^2 + x_2^2 + x_1'^2 + x_2'^2)[\cot(\omega t) + \sqrt{3}\cot(\sqrt{3}\omega t)]$$

$$+ 2(x_1x_2 + x_1'x_2')[\cot(\omega t) - \sqrt{3}\cot(\sqrt{3}\omega t)] - 2(x_1x_1' + x_2x_2')[1/\sin(\omega t)$$

$$+ \sqrt{3}/\sin(\sqrt{3}\omega t)] + 2(x_1x_2' + x_2x_1')\ [-1/\sin(\omega t) + \sqrt{3}/\sin(\sqrt{3}\omega t)]\} \quad . \tag{4.18}$$

5. Propagator for driven coupled harmonic oscillators

When we set $f_1(t) = f_2(t) = 0$, DCHO reduces to CHO, whereby we can write the propagator for DCHO as

$$K(x_1,x_2,t;\ x_1',x_2',0) = \exp[a(t)x_1^2 + b(t)x_1x_2 + c(t)x_2^2 + d(t)x_1$$

$$+ g(t)x_2 + h(t)] \quad . \tag{5.1}$$

Here a(t), b(t), c(t), d(t), f(t) and h(t) are time-dependent functions including x_1' and x_2', which need to be determined. Equation (5.1) must satisfy the Schroedinger equation

$$i\hbar(\partial K/\partial t) = H\ K \tag{5.2}$$

Substitution of Eq. (5.1) into Eq. (5.2) gives the time-dependent coefficients

$$a(t) = \frac{i\hbar}{2m}\,[4a^2(t) + c^2(t)] + m\omega^2/i\hbar \tag{5.3}$$

$$b(t) = \frac{i\hbar}{2m}\,[4b^2(t) + c^2(t)] + m\omega^2/i\hbar \tag{5.4}$$

$$c(t) = \frac{2i\hbar}{m}\,[a(t)c(t) + b(t)c(t)] - m\omega^2/i\hbar \tag{5.5}$$

$$d(t) = \frac{i\hbar}{m}\,[2a(t)d(t) + c(t)g(t)] + (\frac{i}{\hbar})f_1(t) \tag{5.6}$$

$$g(t) = \frac{i\hbar}{m}\,[2b(t)g(t) + c(t)d(t)] + (\frac{i}{\hbar})f_2(t) \tag{5.7}$$

$$h(t) = \frac{i\hbar}{2m}\,[d^2(t) + g^2(t) + 2a(t) + 2b(t)] \quad . \tag{5.8}$$

Since Eqs. (5.3) and (5.4) have the same form, we get

$$a(t) = b(t) \quad . \tag{5.9}$$

Substituting Eq. (5.9) into Eq. (5.5) and changing the variables a and c into

$$\eta = a + c/2 \quad , \tag{5.10}$$

$$\zeta = a - c/2 \quad , \tag{5.11}$$

we obtain two ordinary differential equations:

$$\dot{\eta} = \frac{2i\hbar}{m}\eta^2 + \frac{m\omega^2}{2i\hbar} \tag{5.12}$$

$$\dot{\zeta} = \frac{2i\hbar}{m}\zeta^2 + \frac{3m\omega^2}{2i\hbar} \tag{5.13}$$

The solutions of Eqs. (5.12) and (5.13) are given by

$$\eta = \frac{i\omega m}{2\hbar}\cot(\omega t + \theta_1) \tag{5.14}$$

$$\zeta = \frac{\sqrt{3}i\omega m}{2\hbar}\cot(\sqrt{3}\omega t + \theta_2) \quad , \tag{5.15}$$

where θ_1 and θ_2 are the constants to be determined. The time-dependent coefficients a(t), b(t) and c(t) obtained in comparison with Eqs. (5.10), (5.11), (5.14) and (5.15) are given as

$$a(t) = b(t) = \frac{i\omega m}{4\hbar}\left[\cot(\omega t + \theta_1) + \sqrt{3}\cot(\sqrt{3}\omega t + \theta_2)\right] \tag{5.16}$$

$$c(t) = \frac{i\omega m}{2\hbar}\left[\cot(\omega t + \theta_1) - \sqrt{3}\cot(\sqrt{3}\omega t + \theta_2)\right] \quad . \tag{5.17}$$

Equations (5.16) and (5.17) do not include the driven forces $f_1(t)$ and $f_2(t)$. Therefore, through setting $f_1(t) = f_2(t) = 0$, Eqs. (5.16) and (5.17) do not change at all and should be equal to the coefficients of x_1^2 and x_2^2 in Eq. (4.18). Comparison of these two equations shows θ_1 and θ_2 to be zero. Substituting Eq. (5.9) into Eqs. (5.6) and (5.7) and changing variables d and g into

$$\rho = d + g \tag{5.18}$$

$$\sigma = d - g \quad , \tag{5.19}$$

we obtain the two differential equations

$$\dot{\rho} = \frac{i\hbar}{m}[2a(t) + c(t)]\ \rho + \frac{i}{\hbar}[f_1(t) + f_2(t)] \quad , \tag{5.20}$$

$$\dot{\sigma} = \frac{i\hbar}{m}[2a(t) + c(t)]\ \sigma + \frac{i}{\hbar}[f_1(t) - f_2(t)] \quad . \tag{5.21}$$

Combining Eqs. (5.20) and (5.21) with Eqs. (5.16) and (5.17), we obtain the solutions

$$\rho = [1/\sin(\omega t)]\ \{\int_0^t d\tau\ \frac{i}{\hbar}\ [f_1(\tau) + f_2(\tau)]\ \sin(\omega t) + \alpha\} \tag{5.22}$$

$$\sigma = [1/\sin(\sqrt{3}\omega t)]\ \{\int_0^t d\tau\ \frac{i}{\hbar}\ [f_1(\tau) - f_2(\tau)]\ \sin(\sqrt{3}\omega t) + \beta\} \tag{5.23}$$

where α and β are constants to be determined. We can obtain the time-dependent coefficients d(t) and g(t) by substituting Eqs. (5.22) and (5.23) into Eqs. (5.18) and (5.19):

$$\begin{aligned} d(t) = {} & [i/2\hbar\sin(\omega t)] \int_0^t d\tau[f_1(\tau) + f_2(\tau)]\ \sin(\omega\tau) \\ & + [i/2\hbar\sin(\sqrt{3}\omega t)] \int_0^t d\tau[f_1(\tau) - f_2(\tau)]\ \sin(\sqrt{3}\omega\tau) \\ & + [\alpha/2\sin(\omega t)] + [\beta/2\sin(\sqrt{3}\omega t)] \end{aligned} \tag{5.24}$$

$$g(t) = \left[i/2\hbar\sin(\omega t)\right] \int_0^t d\tau\ [f_1(\tau) + f_2(\tau)]\ \sin(\omega\tau)$$
$$- \left[i/2\hbar\sin(\sqrt{3}\omega t)\right] \int_0^t d\tau\ [f_1(\tau) - f_2(\tau)]\ \sin(\sqrt{3}\omega\tau)$$
$$+ [\alpha/2\sin(\omega t)] - [\beta/2\sin(\sqrt{3}\omega t)] \quad . \tag{5.25}$$

Substitution of Eqs. (5.16), (5.17), (5.24) and (5.25) into Eq. (5.8) yields

$$h(t) = -\frac{i\hbar}{4m\omega}\ [\alpha^2\cot(\omega t) + (\beta^2/\sqrt{3})\ \cot(\sqrt{3}\omega t)]$$
$$- [\alpha/m\omega\sin(\omega t)] \int_0^t d\tau\ [f_1(\tau) + f_2(\tau)]\ \sin[\omega(t-\tau)]$$
$$- [\beta/\sqrt{3}m\omega\sin(\sqrt{3}\omega t)] \int_0^t d\tau\ [f_1(\tau) - f_2(\tau)]\ \sin[\sqrt{3}\omega(t-\tau)]$$
$$+ [1/4i\hbar m\omega\sin(\omega t)] \int_0^t d\tau \int_0^t d\nu\ [f_1(\tau) + f_2(\tau)]$$
$$\times [f_1(\nu) + f_2(\nu)]\ \sin[\omega(t-\tau)]\sin(\omega\nu)$$
$$+ [1/4\sqrt{3}i\hbar m\omega\ \sin(\sqrt{3}\omega t)] \int_0^t d\tau \int_0^t d\nu\ [f_1(\tau) - f_2(\tau)]$$
$$\times [f_1(\nu) - f_2(\nu)]\sin[\sqrt{3}\omega(t-\tau)]\ \sin(\sqrt{3}\omega\nu) - \ell n[\sin(\omega t)$$
$$\times \sin(\sqrt{3}\omega t) + \delta \quad . \tag{5.26}$$

Here, δ is also a constant to be determined. When setting $f_1(t) = f_2(t) = 0$, Eqs. (5.24) and (5.25) should be reduced to the coefficients of x_1 and

x_2, and Eq. (5.26) should also be reduced to the terms in the exponent in Eq. (4.18). Comparison between them gives the constants α, β and δ:

$$\alpha = \frac{m\omega}{i\hbar} (x_1' + x_2') \quad , \tag{5.27}$$

$$\beta = \frac{m\omega}{i\hbar} (x_1' - x_2') \quad , \tag{5.28}$$

$$\delta = \ell n(\frac{3^{\frac{1}{2}} m\omega}{2\pi i\hbar}) \quad . \tag{5.29}$$

Substitution of the above results into Eq. (5.1) gives the propagator for DCHO:

$$K(x_1,x_2,t;\ x_1',x_2',0) = \frac{m\omega}{2\pi i\hbar}\left\{\sqrt{3}/[\sin(\omega t)\ \sin(\sqrt{3}\omega t)]\right\}^{\frac{1}{2}}$$

$$\times \exp\left\{\frac{im\omega}{4\hbar}\left[(x_1^2 + x_2^2 + x_1'^2 + x_2'^2)[\cot(\omega t) + \sqrt{3}\cot(\sqrt{3}\omega t)]\right.\right.$$

$$+ 2(x_1x_2 + x_1'x_2')[\cot(\omega t) - \sqrt{3}\cot(\sqrt{3}\omega t)] - 2(x_1x_1' + x_2x_2')[1/\sin(\omega t)$$

$$+ \sqrt{3}/\sin(\sqrt{3}\omega t)]$$

$$+ 2(x_1x_2' + x_1'x_2)[-1/\sin(\omega t) + \sqrt{3}/\sin(\sqrt{3}\omega t)]$$

$$+ \frac{2x_1}{m\omega}\{[1/\sin(\omega t)] \int_0^t d\tau\ [f_1(\tau) + f_2(\tau)]\ \sin(\omega\tau)$$

$$+ [1/\sin(\sqrt{3}\omega t)] \int_0^t d\tau\ [f_1(\tau) - f_2(\tau)]\ \sin(\sqrt{3}\omega\tau)\}$$

$$+ \frac{2x_2}{m\omega} \{[1/\sin(\omega t)] \int_0^t d\tau \; [f_1(\tau) + f_2(\tau)] \; \sin(\omega\tau)$$

$$- [1/\sin(\sqrt{3}\omega t)] \int_0^t d\tau \; [f_1(\tau) - f_2(\tau)] \; \sin(\sqrt{3}\omega\tau)\}$$

$$+ \frac{4x_1'}{m\omega} \{[1/\sin(\omega t)] \int_0^t d\tau \; [f_1(\tau) + f_2(\tau)] \; \sin[\omega(t-\tau)]$$

$$+ [1/\sin(\sqrt{3}\omega t)] \int_0^t d\tau \; [f_1(\tau) - f_2(\tau)] \; \sin(\sqrt{3}\omega\tau)\}$$

$$+ \frac{4x_2'}{m\omega} \{[1/\sin(\omega t)] \int_0^t d\tau \; [f_1(\tau) + f_2(\tau)] \; \sin[\omega(t-\tau)]$$

$$- [1/\sin(\sqrt{3}\omega t)] \int_0^t d\tau \; (f_1(\tau) - f_2(\tau)) \; \sin(\sqrt{3}\omega\tau)\}$$

$$- [1/m^2\omega^2 \sin(\omega t)] \int_0^t d\tau \int_0^t d\nu \; [f_1(\tau) + f_2(\tau)][f_1(\nu) + f_2(\nu)]$$

$$\times \sin[\omega(t-\tau)] \; \sin(\omega\nu)$$

$$- [1/\sqrt{3}m^2\omega^2 \sin(\sqrt{3}\omega t)] \int_0^t d\tau \int_0^t d\nu \; [f_1(\tau) - f_2(\tau)][f_1(\nu) - f_2(\nu)]$$

$$\times \sin[\sqrt{3}\omega(t-\tau)] \; \sin(\sqrt{3}\omega\nu)\}\Big]\Big\} \; . \qquad (5.30)$$

6. Energy expectation values of coupled harmonic oscillators

The Hamiltonian of CHO is

$$H = \frac{1}{2m}(p_1^2 + p_2^2) + m\omega^2(x_1^2 - x_1x_2 + x_2^2) \quad . \tag{6.1}$$

Using Eqs. (3.1) and (3.2) with Eq. (6.1), we obtain the Schroedinger equation,

$$i\hbar(\partial/\partial t)\ \psi(x_1,x_2,t) = H_{op}\ \psi(x_1,x_2,t) \quad , \tag{6.2}$$

where H_{op} is the Hamiltonian operator in which the momentum p_i is changed into $p_i = (\hbar/i)(\partial/\partial x_i)$. Since Eq. (6.2) can be separated into time and coordinate parts, we may write

$$K(t) = e^{-iH_{op}t/\hbar} \tag{6.3}$$

$$H_{op}|\ell,n\rangle = E_{\ell n}|\ell,n\rangle \quad . \qquad (\ell,n = 1,2,3,\ldots) \tag{6.4}$$

Here the states $|\ell,n\rangle$ are the complete set with energy eigenvalues of H_{op}. Since the function with states $|\ell,n\rangle$ can be expressed by

$$\phi_{\ell n}(x_1,x_2) = \langle x_1,x_2|\ell,n\rangle \quad , \tag{6.5}$$

the propagator at $t > 0$ becomes

$$K(x_1,x_2,t;\ x_1',x_2',0) = \langle x_1,x_2|e^{-iH_{op}t/\hbar}|x_1',x_2'\rangle$$

$$= \sum_{\ell} \sum_{n} \sum_{\ell'} \sum_{n'} \langle x_1, x_2 | \ell n \rangle \langle \ell n | e^{-iH_{op} t/\hbar} | \ell', n' \rangle \langle \ell', n' | x_1', x_2' \rangle$$

$$= \sum_{\ell} \sum_{n} \phi_{\ell n}(x_1, x_2)\, e^{-iE_{\ell n} t/\hbar}\, \phi^*_{\ell n}(x_1', x_2') \quad . \tag{6.6}$$

Equation (6.6) should be the same as Eq. (4.18). Setting $x_1' = x_1$ and $x_2' = x_2$ in Eq. (4.18) and integrating over x_1 and x_2, we get

$$\sum_{\ell} \sum_{n} \int \int dx_1 dx_2\, \phi^*_{\ell n}(x_1, x_2)\, e^{-iE_{\ell n} t/\hbar}\, \phi_{\ell n}(x_1, x_2) = e^{-iE_{\ell n} t/\hbar} \tag{6.7}$$

and

$$\int \int dx_1 dx_2\, \frac{m\omega}{2\pi i\hbar} \left\{ \sqrt{3}/[\sin(\omega t)\sin(\sqrt{3}\omega t)] \right\}^{\frac{1}{2}}$$

$$\times \exp\{\frac{im\omega}{2\hbar}[(x_1 + x_2)^2 - \sqrt{3}(x_1 - x_2)^2][\cot(\omega t) - 1/\sin(\omega t)]\}$$

$$= - \frac{1}{2} [\sin(\omega t/2)\sin(\sqrt{3}/2\omega t)]^{-1} \quad . \tag{6.8}$$

Hence, we have

$$\sum_{\ell} \sum_{n} e^{-iE_{\ell n} t/\hbar} = - \frac{1}{2} [\sin(\omega t/2)\sin(\sqrt{3}/2\omega t)]^{-1}$$

$$= [e^{-i\omega t/2}/(1 - e^{-i\omega t})][e^{-i\sqrt{3}\omega t/2}/(1 - e^{-i\sqrt{3}\omega t})]$$

$$-\sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} \exp\{-i\omega t[(\ell + \frac{1}{2}) + \sqrt{3}(n + \frac{1}{2})]\} \quad . \qquad (6.9)$$

Therefore the expectation values of CHO becomes

$$E_{\ell n} = [(\ell + \frac{1}{2}) + \sqrt{3}(n + \frac{1}{2})] \hbar\omega \quad . \qquad (6.10)$$

7. **Results and discussion**

In the previous sections we have obtained the exact propagators [Eqs. (4.18) and (5.30)] for CHO and DCHO by the path-integral method. The forms of the propagators are new. Setting $f(t) = 0$, Eq. (5.30) is reduced to Eq. (4.18). Although DCHO is a nonconservative system, the quantum-mechanical problem for the momentum operator does not appear because the canonical momentum is equal to the kinetic momentum in our derivation.[13]

Making use of Eq. (4.18), we have obtained the energy expectation values [Eq. (6.10)] for CHO, given by the sum of two energy expectation values corresponding to the quantum states of two oscillators. Even though we have not evaluated the wavefunction of CHO, we may easily surmise that the wavefunction will be given by the multiplication of two wavefunctions for two oscillators. In the case of DCHO, one cannot easily apply Eq. (5.20) to obtain the energy expectation values, since this equation cannot be expressed in the form of Eq. (6.6), and one should recognize that the energy operator is not equal to the Hamiltonian operator in a nonconservative system.[9]

The evaluations for the wavefunctions, energy expectation values for CHO and DCHO, and propagator and other physical quantities for n coupled and n driven coupled harmonic oscillators (arbitrary n) are in progress and will be reported in the near future.

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